

RASCH MEASUREMENT

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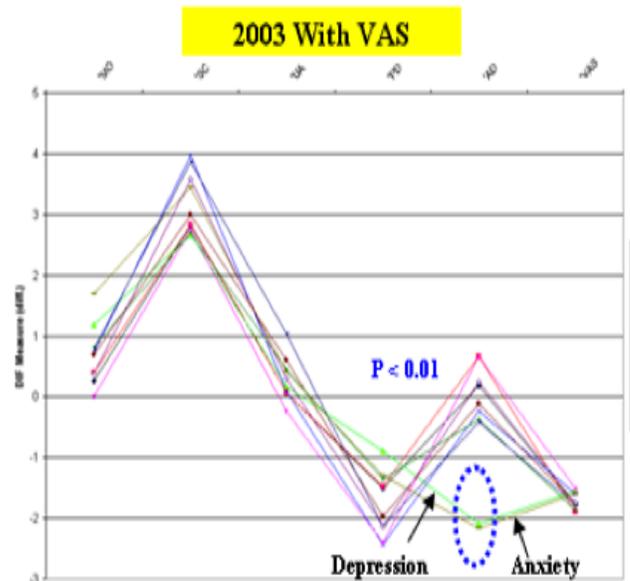
ISSN 1051-0796

(EQ-5D + VAS) x Rasch = HRQoL Measure

The *EQ-5D* questionnaire and the EQ Visual Analogue Scale (*EQ-VAS*) were developed by the EuroQol Group for deriving the preference-weighted single indices used for economics studies involving tradeoffs. This study investigates the extent to which the five EQ-5D items and the VAS together form a valid measure of Health Related Quality of Life (HRQoL) in a U.S. representative sample diagnosed with the most prevalent chronic health conditions.

The *EQ-5D-3L* has 5 items classifying health in terms of mobility (MO), self-care (SC), usual activities (UA), pain/discomfort (PD) and anxiety/depression (AD), with the 3 response category coding gave lower scores for healthier respondents. The original 101 *VAS* categories were collapsed to form a 9-category item to ensure sufficient frequency of endorsement for each VAS category with coding reversed to be consistent with the EQ-5D.

Respondents extracted from the 2-year panel (2002-03) from the Medical Expenditure Panel Survey (MEPS) 1) were ≥ 18 years of age; 2) had complete EQ-5Ds, and 3) reported primary ICD-9-CM for the top 10 most prevalent chronic health conditions (Table 1).



The Rasch rating scale model (RSM) was used to calibrate the responses on the five items and the partial credit model (PCM) for the categorized VAS scores. Fit of the six items to the Rasch model was assessed by using the INFIT mean square (INFIT MNSQ < 1.4).

Figures: Gender-related DIF with VAS - 2002, and Disease-related DIF with VAS - 2003. The x-axis shows the EQ-5D items and the y-axis is the DIF measure by DIF grouping. ($p < 0.01$).

Results: There was significant *gender-related DIF* ($p < 0.01$) on the EQ 5D “anxiety/depression” mental health item and significant *disease-related DIF* ($p < 0.01$) “anxiety/depression” for respondents diagnosed with depression or anxiety, with or without the VAS, in both

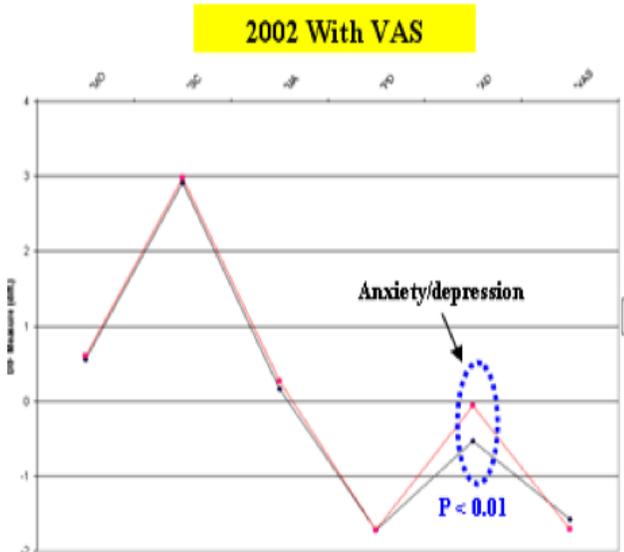


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years. Responses from the VAS consistently fit the Rasch model (INFIT MNSQ < 1.4) over the two time points and across all ten disease groups.

Misfit was shown by the “*anxiety/depression*” mental health item, esp. with 2002 data; and improved fit in 2003, esp. *with* the inclusion of VAS. Analyses using a gender-split on the “*anxiety/depression*” item improved fit to the model among males (in 8 out of 10 first year and 9 out of 10 second year disease groups). When enhanced by the inclusion of the VAS, the EQ-5D descriptive system exhibited satisfactory Rasch measurement qualities and further enhancement was achieved by purging the gender and disease effects.

Results compared across both years revealed the crucial measurement property of invariance for the EQ-5D. The findings suggest that 1) the EQ-5D descriptive system and the EQ-VAS can be combined together to provide an overall measure of HRQoL and, 2) together they might serve as a suitable *measurement* framework for deriving population preference-weights.

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Trevor G. Bond, School of Education, James Cook University, Townsville, Queensland, Australia

Benjamin M. Craig, Health Outcomes and Behaviors, Moffitt Cancer Center, Tampa, Florida USA

Reference: The EuroQol group (1990) “EuroQol a new facility for the measurement of health related quality of life.” *Health Policy*, 16,199-208.

Based on a poster presented at 26th Plenary meeting of the EuroQol Group , Paris, France, September 3rd - 5th, 2009

Table 1. Respondents (N = 2,057)					
Mean age (SD) 52.05 (16.53)					
	n	%		n	%
Race			Disease		
White	1610	78.27	Hypertension	336	23.53
Black	337	16.38	Diabetes	215	15.06
Asian	57	2.77	Arthropathy	170	11.90
Other	53	2.58	Depression	168	11.76
VAS			Back Disorder	159	11.13
0-20	49	2.38	Joint Disorder	80	5.60
21-30	50	2.43	Chronic sinusitis	79	5.53
31-40	75	3.65	Anxiety	74	5.18
41-50	169	8.22	Asthma	74	5.18
51-60	137	6.66	Cholesterol	73	5.11
61-70	244	11.86			

RUMM2020 to RUMM2030

An enhanced RUMM application for conducting Rasch item analyses was released as RUMM2030 in January 2010.

RUMM2030 offers a further advancement in the conduct of interactive Rasch analysis within the Rasch paradigm of measurement. It is available in two editions: *Standard* and *Licence*.

The *Standard Edition* of RUMM2030 is an upgrade from RUMM2020 in its functions and presentation. It replaces RUMM2020, which was released in 2003. It is available as a one-time purchase.

Major additional features of the Standard edition include:

1. creating data sets with complete data records only (in the case of random missing data).
2. additional details in test equating.
3. the addition of Person Characteristic Curves and standard residual plots for improving the diagnostic feedback on both the fit and response patterns of individual person responses across items attempted.
4. reference of the cumulative person distribution to the normal counterpart.
5. formalising the similarity and distinctions between the traditional reliability index, determined by Cronbach’s Alpha, and the Person Separation index, especially in relation to person-item targeting.

The *Licence Edition* of RUMM2030 is an expansion of the Standard edition. These additions are a product of many years of research. It is available on an upgrade/maintenance basis and is purchased as a renewable licence for a specified period of time. The licence will include the latest upgrade with each renewal. The licence will help research and ensure that the improvements of RUMM will continue to enhance the Rasch paradigm of measurement. If the Licence edition is not renewed, RUMM2030 will revert back to the Standard edition.

Major additional features of the Licence edition include:

1. assessment of dimensionality.
2. assessment of local response dependence.
3. conditional test-of-fit for a pair of polytomous items or a pair of tests.
4. post hoc tailored response analysis, e.g., for testing the significance of guessing.
5. Facet Analysis for up to a 3-way item response structure.

For more about RUMM2030:

www.rummlab.com.au

IOMW 2010: The 15th International Objective Measurement Workshop
Wednesday, April 28th and Thursday, April 29, 2010
(Immediately preceding AERA and NCME Conferences, April 30 - May 4, 2010)
Boulder, Colorado, USA

Call for Proposals: Deadline - Feb. 5, 2010

Conference website: www.iomw2010.net

Theme: Using Model Fit to Evaluate Hypotheses about Learning

The International Objective Measurement Workshop is a biennial conference devoted to the presentation and discussion of topics germane to the theory and practice of measurement. This year's conference theme focuses attention upon the evaluation of model fit as a means of improving our understandings of measurement constructs. The evaluation of fit in item response theory is often either not well understood or not given sufficient scrutiny. Casual rules of thumb for fit statistic interpretations are sometimes followed that may mask the presence of unusual—and revealing—response patterns. Furthermore, fit is often evaluated at the item level without giving equal scrutiny to fit at the person level. Beyond these problems, even when patterns of “misfit” have been identified, the proper response is often equivocal. The explanation for misfit could be found in the use of misaligned assessment items, a faulty hypothesis of how growth along the construct develops, or unusual characteristics in the makeup of the sample of students for whom empirical evidence has been collected.

One specific context in which rigorous evaluations of model fit are needed is in the measurement of learning progressions. Learning progressions are descriptions of increasingly sophisticated ways of thinking about or understanding a topic. One of the more appealing features of learning progressions is their potential use to facilitate diagnostic assessments of student understanding over time. In the context of learning progressions, it is important to rethink expectations in applying a measurement model to item responses. Modelers are apt to declare success when the difference between what is predicted and what is observed seems rather small. Yet in the case of assessment tasks that stem from a learning progression hypothesis, developers should not only be prepared to find considerable evidence of misfit, they should embrace it and use this as a means to revise and improve their instrumentation. Presentation proposals to the IOMW conference are encouraged that focus on these sorts of issues.

The conference will be held at the University Memorial Center at the University of Colorado in Boulder, Colorado, www.colorado.edu, which is approximately a half an hour drive from Denver and the AERA Conference Hotels. For details on proposal submission and conference registration, please visit www.iomw2010.net

REGISTRATION: \$40 (Students \$30) [after 2/5/10 registration fee increases by \$10]

LODGING: www.iomw2010.net/lodging.html

Journal of Applied Measurement

Volume 10, Number 4. Winter 2009

The Rasch Model and Additive Conjoint Measurement. *Van A. Newby, Gregory R. Conner, Christopher P. Grant, and C. Victor Bunderson. p. 348-354.*

The Construction and Implementation of User-Defined Fit Tests for Use with Marginal Maximum Likelihood Estimation and Generalized Item Response Models. *Raymond J. Adams and Margaret L. Wu. p. 355-369.*

Development of a Multidimensional Measure of Academic Engagement. *Kyra Caspary and Maria Veronica Santelices. p. 371-393.*

Random Parameter Structure and the Testlet Model: Extension of the Rasch Testlet Model. *Insu Paek, Haniza Yon, Mark Wilson, and Taehoon Kang. p. 394-407.*

A Comparative Analysis of the Ratings in Performance Assessment Using Generalizability Theory and Many-Facet Rasch Model. *Sungsook C. Kim and Mark Wilson. p. 408-423.*

The Family Approach to Assessing Fit in Rasch Measurement. *Richard M. Smith and Christie Plackner. p. 424-437.*

Understanding Rasch Measurement: Standard Setting with Dichotomous and Constructed Response Items: Some Rasch Model Approaches. *Robert G. MacCann. p. 438-454.*

Richard M. Smith, Editor

JAM web site: www.jampress.org

First International IACAT Conference on Computerized Adaptive Testing

7-9 June, Arnhem, The Netherlands

Deadline for Submissions of Proposals: February 15, 2010

The *International Association for Computerized Adaptive Testing* (IACAT) is a nascent organization dedicated to advancing computerized adaptive testing (CAT) through research and education. IACAT will hold its first annual conference June 7- 9, 2010. The conference will take place in Arnhem, The Netherlands. The conference, hosted by the Research Center for Examination and Certification (RCEC: www.rcec.nl), will take place at the Conference Centre Papendal. (www.papendal.com)

Program

The conference will be an international forum for CAT researchers and others interested in CAT to meet and share ideas and developments on CAT. At the conference, the following well-known scholars will give keynote presentations:

- Cees Glas, University of Twente, The Netherlands
- Mark Reckase, Michigan State University, USA
- Lawrence Rudner, Graduate Management Admission Council, USA
- Wim van der Linden, CTB/McGraw-Hill, USA
- Otto Walter, Institute of Psychology, Aachen University, Germany
- Matthew Finkelman, Tufts University School of Dental Medicine, USA

The conference will include paper and poster presentations for which proposals are welcomed. There will also be time for informal meetings and social activities.

Registration

Registration is open now at our website, www.rcec.nl/iacat by filling in the registration forms.

Pre-conference workshops

On June, 7 from 9.00-12.00, workshops are organized on introductory and more advanced topics in CAT.

- Nathan Thompson, Assessment Systems Corporation: Introduction to CAT
- Mark Reckase, Michigan State University: Multidimensional CAT
- Bernard Veldkamp; University of Twente: Item selection in CAT

Proposals

IACAT welcomes proposals for conference presentations at this time. Presentations will be in one of two formats: a 20-minute session and a traditional poster format. Proposals must include title, authors, and a description of 250 words or less. CAT research generally falls into two categories: (1) theoretical research on algorithms and (2) applications and implementations of CAT. Both types of research are welcome at the conference; please also specify the category when submitting. Proposals will be evaluated on technical quality, practical applicability, advancement of knowledge, and perceived interest to participants.

Proposals can be submitted by filling in the proposal submission form on the conference web site: www.rcec.nl/iacat

The deadline for submission is **February 15, 2010**. Proposals will be reviewed and notice of acceptance will be given before April, 1, 2010.

Organization

The conference is organized by a committee consisting of:

- Clifford Donath, Donath Group, USA
- Theo Eggen, Cito, University of Twente, Netherlands
- Nathan Thompson, Assessment Systems Corporation, USA
- Davis Weiss, University of Minnesota, USA
- Birgit Olthof, RCEC, University of Twente, Netherlands

About IACAT

Previous CAT conferences were held in 2007 and 2009, sponsored by the Graduate Management Admissions Council. IACAT was founded at the 2009 conference, and the 2010 conference marks the first official function of the organization. Come take part in history! To learn more about IACAT or to join, please visit www.iacat.org

Advanced course in Rasch Measurement of Modern Test Theory

The University of Western Australia (UWA) - Graduate School of Education

ONLINE COURSE

Semester 1, 22 February to 18 June 2010

Professor David Andrich and Dr Ida Marais

www.education.uwa.edu.au/pp/courses

This course is intended as a follow-up to the *'Introduction to Rasch Measurement of Modern Test Theory'* course, also available at the University of Western Australia. It can be studied from anywhere in the world and an online discussion site will operate as part of the course. Students enrolled obtain (i) the study guide; (ii) a set of lecture materials, which includes a hard copy of all of the lectures; (iii) details of the assignments; (iv) selected readings and (v) a copy of the RUMM2030 software for the duration of the course.

Topics covered

- Theory of social measurement and Rasch models
- Revision of Rasch's model for dichotomous responses and his original work
- Multiple choice items and guessing
- Understanding the Polytomous Rasch Model
- Model fit statistics
- Advanced understanding of differential item functioning (DIF): real and artificial DIF
- Vertical equating and DIF
- Assessing two violations of the assumption of local independence: multidimensionality and response dependence
- Facets analysis and analysis of longitudinal data
- Reporting a Rasch analysis

Assignments

Assignments include analyses of real and simulated data sets using RUMM2030 and one assignment where participants have the option of analyzing their own data.

Prof David Andrich (david.andrich ~at~ uwa.edu.au or +61 8 6488 1085)

Dr Ida Marais (ida.marais ~at~ uwa.edu.au or +61 8 6488 3353)

Review of Spitzer's *Transforming Performance Measurement*

Everyone interested in practical measurement applications needs to read Dean R. Spitzer's book, *Transforming performance measurement: Rethinking the way we measure and drive organizational success* (New York, AMACOM, 2007). Spitzer describes how measurement, properly understood and implemented, can transform organizational performance by empowering and motivating individuals. Measurement understood in this way moves beyond quick fixes and fads to sustainable processes based on a measurement infrastructure that coordinates decisions and actions uniformly throughout the organization.

Measurement leadership, Spitzer says, is essential. He advocates, and many organizations have instituted, the C-suite (executive) position of Chief Measurement Officer (Chapter 9). This person is responsible for instituting and managing the four keys to transformational performance measurement (Chapters 5-8):

- Context sets the tone by presenting the purpose of measurement as either negative (to inspect, control, report, manipulate) or positive (to give feedback, learn, improve).
- Focus concentrates attention on what's important, aligning measures with the mission, strategy, and with what needs to be managed, relative to the opportunities, capacities, and skills at hand.
- Integration addresses the flow of measured information throughout the organization so that the covariations of different measures can be observed relative to the overall value created.
- Interactivity speaks to the inherently social nature of the purposes of measurement, so that it embodies an alignment with the business model, strategy, and operational imperatives.

Spitzer takes a developmental approach to measurement improvement, providing a *Measurement Maturity Assessment* in Chapter 12, and also speaking to the issues of the "living company" raised by Arie de Geus' classic book of that title. Plainly, the transformative potential of performance measurement is dependent on the maturational complexity of the context in which it is implemented.

Spitzer clearly outlines the ways in which each of the four keys and measurement leadership play into or hinder transformation and maturation. He also provides practical action plans and detailed guidelines, stresses the essential need for an experimental attitude toward evaluating change, speaks directly to the difficulty of measuring intangible assets like partnership, trust, skills, etc., and shows appreciation for the value of qualitative data.

Transforming Performance Measurement is not an academic treatise, though all sources are documented,

with the end-notes and bibliography running to 25 pages. It was written for executives, managers, and entrepreneurs who need practical advice expressed in direct, simple terms. Further, the book does not include any awareness of the technical capacities of measurement as these have been realized in numerous commercial applications in high stakes and licensure/certification testing over the last 50 years. This can hardly be counted as a major criticism, since no books of this kind have yet to date been able to incorporate the often highly technical and mathematical presentations of advanced psychometrics.

That said, the sophistication of Spitzer's conceptual framework and recommendations make them remarkably ready to incorporate insights from measurement theory, testing practice, developmental psychology, and the history of science. Doing so will propel the strategies recommended in this book into widespread adoption and will be a catalyst for the emerging re-invention of capitalism. In this coming cultural revolution, intangible forms of capital will be brought to life in common currencies for the exchange of value that perform the same function performed by kilowatts, bushels, barrels, and hours for tangible forms of capital (Fisher, 2009, 2010).

Pretty big claim, you say? Yes, it is. Here's how it's going to work.

- First, measurement leadership within organizations that implements policies and procedures that are context-sensitive, focused, integrated, and interactive (i.e., that have Spitzer's keys in hand) will benefit from instruments calibrated to facilitate:
 - o meaningful mapping of substantive, additive amounts of things measured on number lines;
 - o data volume reductions on the order of 80-95% and more, with no loss of information;
 - o organizational and individual learning trajectories defined by hierarchies of calibrated items;
 - o measures that retain their meaning and values across changes in item content;
 - o adapting instruments to people and organizations, instead of vice versa;
 - o estimating the consistency, and the leniency or harshness, of ratings assigned by judges evaluating performance quality, with the ability to remove those effects from the performance measures made;
 - o adjusting measurement precision to the needs of the task at hand, so that time and resources are not wasted in gathering too much or too little data; and

- o providing the high quality and uniform information needed for networked collective thinking able to keep pace with the demand for innovation.
- Second, measurement leadership sensitive to the four keys across organizations, both within and across industries, will find value in:
 - o establishing industry-wide metrological standards defining common metrics for the expression of the primary human, social, and natural capital constructs of interest;
 - o lubricating the flow of human, social, and natural capital in efficient markets broadly defined so as to inform competitive pricing of intangible assets, products, and services; and
 - o new opportunities for determining returns on investments in human, community, and environmental resource management.
- Third, living companies need to be able to mature in a manner akin to human development over the lifespan. Theories of hierarchical complexity and developmental stage transitions that inform the rigorous measurement of cognitive and moral transformations (Dawson & Gabrielian, 2003) will increasingly find highly practical applications in organizational contexts.

Leadership of the kind described by Spitzer is needed not just to make measurement contextualized, focused, integrated, and interactive “and so productive at new levels of effectiveness”, but also to apply systematically the technical, financial, and social resources needed to realize the rich potentials he describes for the transformation of organizations and empowerment of individuals. Spitzer’s program surpasses the usual focus on centralized statistical analyses and reports to demand the organization-wide dissemination of calibrated instruments that measure in common metrics. The flexibility, convenience, and scientific rigor of instruments calibrated to measure in units that really add up fit the bill exactly. Here’s to putting tools that work in the hands of those who know what to do with them!

William P. Fisher, Jr.

Dawson, T. L., & Gabrielian, S. (2003, June). Developing conceptions of authority and contract across the life-span: Two perspectives. *Developmental Review*, 23(2), 162-218.

Fisher, W. P., Jr. (2009, November). Invariance and traceability for measures of human, social, and natural capital: Theory and application. *Measurement (Elsevier)*, 42(9), 1278-1287

Fisher, W. P., Jr. (2010). Additional material available at www.livingcapitalmetrics.com.

Pacific Rim Objective Measurement Symposium

PROMS-KL 2010

Kuala Lumpur, Malaysia

29 June - 1 July 2010 (Tues-Thurs)

www.iiu.edu.my/proms/2010

Deadline for submissions: 20 April 2010

28 June 2010 (Mon) - Pre-conference half-day workshops

2 July 2010 – (Fri) Trip to Melaka (Malacca)

3 - 4 July 2010 (Sat-Sun) two day Rasch workshops

In 2005, the Research Centre of the International Islamic University Malaysia (IIUM) held the first Pacific Rim Objective Measurement Symposium (PROMS). Since then, PROMS has been held in other regions of the Pacific Rim: Hong Kong (2006), Taiwan (2007), Tokyo (2008), and again recently in Hong Kong in 2009. We are pleased to announce that PROMS 2010 has come home to Kuala Lumpur, Malaysia and is hosted by the Institute of Education, IIUM.

PROMS 2010 KL, as with the previous PROMS, focuses on recent advances of objective measurement. It aims to provide an international forum for discourse on the latest research in using Rasch measurement as a tool for scientific progress. It is our pleasure to invite you to join PROMS panel of distinguished researchers and other practitioners to share their expertise and your experiences in objective measurement.

Unlike the other PROMS meetings, PROMS 2010 KL also invites paper/poster presentations on various issues utilizing other methodologies and approaches to measurement. Parallel sessions for these non Rasch-based papers have been arranged to encourage greater participation from the academic and research community.

PROMS 2010 KL is especially beneficial to postgraduate students and researchers who seek to use the Rasch Measurement Model in their research as experts from the Pacific Rim, Europe, and the United States make it a point to convene at PROMS meetings. Pre-conference and post-conference workshops on Rasch Measurement software applications and special topics will also be given by experts in the field.

More information at www.iiu.edu.my/proms/2010

Dr. Noor Lide Abu Kassim
PROMS-KL 2010 organizer

Stimulating Excellence in Education

Comments on: Stimulating Excellence: Unleashing the Power of Innovation in Education

May 2009, The Center for American Progress et al., www.americanprogress.org/issues/2009/05/entrepreneurs_event.html

The report focuses on creating the conditions for entrepreneurial innovation and reward in education. It deplores the lack of a quality improvement culture in education, and the general failure to recognize the vital importance of measuring performance for active management. It makes specific recommendations aimed at drastic improvements in information quality. *Excellent so far!* But the report would be far more powerful and persuasive if it capitalized on two very significant features of the current situation.

First, only on page 34, in the report's penultimate paragraph, do the authors briefly touch on what all educators know is absolutely the most important thing to understand about teaching and learning: **it always starts from where the student is at, growing out of what is already known.** This is doubly important in the context of the report's focus, teaching and learning about how to institute a new culture of power metrics and innovation. To try to institute fundamental changes, with little or no concern for what is already in place, is a sure recipe for failure.

Second, there is one feature of the educational system as it currently exists that will be of particular value as we strive to improve the quality of the available information. That feature concerns tests and measurement. Many of the report's recommendations would be quite different if its authors had integrated their entrepreneurial focus with the **technical capacities of state-of-the-art educational measurement.**

The obvious recommendation with which to start concerns the reason why public education in the United States is such a fragmented system: because outcome standards and product definitions are expressed (almost) entirely in terms of locally-determined content and expert opinion. Local content, standards, and opinions are essential, but to be meaningful, comparable, practical, and scientific they have to be brought into a common scale of comparison.

The technology for creating such scales is widely available. For over 40 years, commercial testing agencies, state departments of education, school districts, licensure and certification boards, and academic researchers have been developing and implementing stable metrics that transcend the local particulars of specific tests. The authors of the "Stimulating Excellence" report are right to stress the central importance of comparable measures in creating an entrepreneurial environment in education, but they did not do enough to identify existing measurement capabilities and how they could help create that environment.

For instance, all three of the recommendations made at the bottom of page 12 and top of page 13 address

capabilities that are already in place in various states and districts around the country. The examples that come easiest to mind involve the *Lexile Framework for Reading and Writing*, and the *Quantile Framework for Mathematics*, developed by MetaMetrics, Inc., of Durham, NC (www.lexile.com).

The Lexile metric for reading ability and text readability unifies all major reading tests in a common scale, and is used to report measures for over 28 million students in all 50 states. Hundreds of publishers routinely obtain Lexile values for their texts, with over 115,000 books and 80 million articles (most available electronically) Lexiled to date.

Furthermore, though one would never know from reading the "Stimulating Excellence" report, materials on the MetaMetrics web site show that the report's three recommendations concerning the maximization of data utility have already been recognized and acted on, since

- many standardized assessments are already aligned with state learning standards,
- available products already quickly incorporate assessment results into the process of teaching and learning (and a lot more quickly than "a day or two after testing"!), and
- several states already have years of demonstrated commitment to keeping their standards and assessments relevant to the changing world's demands on students.

That said, a larger issue concerns the need to create standards that remain invariant across local specifics. A national curriculum and national testing standards seem likely to fall into the trap of either dictating specific content or fostering continued fragmentation when states refuse to accept that content. But in the same way that computer-adaptive testing creates a unique examination for each examinee "without compromising comparability" so, too, must we invest resources in devising a national system of educational standards that both takes advantage of existing technical capabilities and sets the stage for improved educational outcomes.

That is what the report's key recommendation ought to have been. An approximation of it comes on page 35, with the suggestion that now is the time for investment in what is referred to as "backbone platforms" like the Internet. Much more ought to have been said about this, and it should have been integrated with the previous recommendations, such as those concerning information quality and power metrics. For instance, on page 27, a recommendation is made to "build on the open-source concept." Upon reading that, my immediate thought was that the authors were going to make an analogy with

adaptively administered item banks, not literally recommend actual software implementation processes.

But they took the literal road and missed the analogical boat. That is, we ought to build on the open-source concept by creating what might be called crowd-sourced “wikitests” exams that teachers and researchers everywhere can add to and draw from, with the qualification that the items work in practice to measure what they are supposed to measure, according to agreed-upon data quality and construct validity standards. This process would integrate local content standards with global construct standards in a universally uniform metric not much different from the reference standard units of comparison we take for granted in measuring time, temperature, distance, electrical current, or weight. Michael K. Smith suggests a practical approach to achieving these objectives in “Why not a national test for everyone?”, *Phi Delta Kappan*, 91, 4, Feb. 2010, 54-58.

And this is where the real value of the “backbone platform” concept comes in. The Internet, like phones and faxes before it, and like alphabetic, phonetic and grammatical standards before them, provides the structure of common reference standards essential to communication and commerce. What we are evolving toward is a new level of complexity in the way we create the common unities of meaning through which we achieve varying degrees of mutual understanding and community.

In addition, measurement plays a fundamental role in the economy as the primary means of determining the relation of price to value. The never-ending spiral of increasing costs in education is surely deeply rooted in the lack of performance metrics and an improvement culture. We ought to take the global infrastructure of measurement standards as a model for what we need as a “backbone platform” in education. We ought to take the metaphor of transparency and the need for “clear metrics” much more literally. We really do need instruments that we can look right through, that bring the thing we want to see into focus, without having to be primarily concerned with which particular instrument it is we are using.

Decades of research in educational measurement show that these instruments can be constructed. A great deal still needs to be done, and the challenges are huge, but taking them on will enable us to expand the domains in which we insist on fair dealing, and in which the balance scale applies as a symbol of justice.

When the entrepreneurial vision presented in the “Stimulating Excellence” report is situated in a context better informed by what educators are already doing and what they already know, the stage will be set for a new culture of performance improvement in education, a culture that explicitly articulates, tests, and acts on its educational values. At that point, we can expect great things!

William P. Fisher, Jr.

Union SD 6th Grade Mathematics Q1 Benchmark			Distribution of items (n=20)			
Level	Estimate	Distribution of students (n=445)	Algebra and Functions (n=15)			Number Sense (n=5)
			1.1	1.3	1.4	2.3
Advanced	3	XXXXXX				
	2	XXXXXXXXXXXXXX XXXXXXXXXXXXXX XXXXXXXXXXXXXX X		5		13
Proficient	1	XXXXXXXXXX				12
		XXXXXXX XXXXXXXXXXXXXX	4	6		8 14
Basic	0	XXXXXX	11	18		
		XX XX XX	15 16	19	20	
Below Basic	-1	XX	17			
		XX X	2			1
Far Below Basic		X	3 9	10 7		
Each X represents 5 students			Cronbach's alpha = .78			
(---) Average Proficiency			Person Separation Reliability = .67			
Proficiency levels are approximated based on the student proficiency estimates (theta values) published in the CST Technical Report - Spring 2008 Administration. These cut points assume that the items on the benchmark are perfectly aligned with the items on the CST.						

Diana Wilmot, Ph.D., *Coordinator*, Assessment and Accountability, Santa Clara County Office of Education, California

Rasch Lessons from the Netflix® Prize Challenge Competition

The Netflix Prize Challenge Competition ran for 34 months from October 2, 2006 until July 26, 2009. Netflix supplied a “Training” dataset of 100,480,507 ratings made by 480,189 Netflix clients for 17,770 movies between October, 1998 and December, 2005. The ratings were on a rating scale of one star to five stars. The Training data matrix has 99% missing data. Netflix also supplied a dataset of 2,817,131 “Qualifying” ratings. For these ratings, the clients and movies are known, but the actual ratings were known only to Netflix (until the competition concluded). The Netflix Prize was awarded to the team most successful at “predicting” those publicly unknown ratings. Teams were allowed to submit multiple prediction datasets.

Team “BellKor’s Pragmatic Chaos” won the Prize in a tie-break based on a 20-minute earlier submission time of their winning prediction dataset. The runners-up were team “The Ensemble” (of which I was a member). The team that had the “dubious honors” (according to Netflix) of the very worst predictions, out of the 44,014 valid submissions from 5,169 actively participating teams, was team “Lanterne Rouge” of which I was the leading member. Of course, these worst predictions were deliberate!

During the 34 months of the Prize competition, there were some valuable lessons with general application in Rasch analysis.

1. Dataset size.

When stored as a rectangular text data-file, the size of the Training is at least $480,189 * 17,770$ bytes = 8,532,958,530 bytes = 8GB. When implemented in Winsteps, this required 8GB of input data file and two more work files of the same size = 24 GB (at least). But 99% of these 24GB are missing data. So this was highly inefficient. Simultaneously, Winsteps users were noticing the same thing for their computer-adaptive-testing (CAT) and concurrent-test-equating analyses. In contrast, the same data in Facets (for which missing data do not need to be stored) have an input dataset size of 1.3GB and a work file size of 0.6GB. So obvious improvements to Winsteps were to allow Winsteps to use a Facets-style input-data-format, and to use a compressed work-file algorithm. This reduced the Winsteps input dataset size to 1.3GB and the work-file sizes reduced to 3.3GB and 0.2GB, a total of 5GB instead of 24GB.

2. Processing time.

The first run of Winsteps on the Training dataset indicated that the estimation process would take 8 days to come to convergence. Consequently that first run was cancelled after 1 day as entirely impractical. The first run in Facets on the same data required about 24 hours. Again this suggested improvements could be made to Winsteps. Reducing the dataset size also reduced the input-output overhead, so reducing processing time. But inspection of

the computer code also revealed routines which could be made faster. Consequently Winsteps processing time was reduced to about 12 hours, and much less if only rough convergence is required.

3. Rasch Models.

Each time a dataset of predictions was submitted to Netflix, Netflix responded with a summary statistic on the accuracy with which the “Quiz” half of the qualifying ratings had been predicted. Competitors did not know which of the Qualifying ratings comprised the Quiz dataset. The other half of the Qualifying ratings were termed the “Test” dataset. The summary statistic for the Quiz dataset was the root-mean-square-residual (RMSR), called by Netflix the root-mean-square-error (RMSE), between the known-to-Netflix values of the ratings and their competitor-predicted values. The values of the RMSRs enabled competitors to know which of their prediction models were more effective. Netflix permitted submissions to include predictions between categories, such as 3.5674 stars. This improved RMSRs relative to predicting exact categories.

An immediate finding was that the Rasch-Andrich Rating Scale model (RSM), applied to the 5 category (1 star to 5 star) Netflix rating scale was more effective (RMSR=0.9815) than either a Rasch-Masters Partial Credit model applied to the 17,770 movies (RMSR=0.9867) or to the 480,189 clients (RMSR=0.9907). Less parameters, but better prediction!

4. Pseudo-Rasch Dichotomous Fractional Model.

As the competition proceeded, it became apparent that the data were severely multidimensional, and that a unidimensional Rasch analysis was a useful first-stage leading on to other analyses. But, as implemented in Winsteps and Facets, the Andrich Rating Scale model requires the computation of 4 exponentials for each observation in each estimation iteration as well as the accumulation of probabilities for the five categories. Further the threshold estimates need to be brought to convergence. If this processing load could be lessened, without severely impacting the utility of the Rasch measures, then the duration of the first-stage Rasch analysis would be considerably reduced.

This motivated the “Pseudo-Rasch Dichotomous Fractional Model” (DFM). In the Rasch dichotomous model (DM), the observations are “1” (success) and “0” (failure). In RSM, the Netflix rating scale is modeled to be 5 qualitatively-ordered categories along the latent variable. In DFM the 5 categories are modeled to be predictions of the probability of success on a dichotomous item. 5 stars = 1.0 probability. 4 stars = 0.75 probability. 3 stars = 0.5 probability. 2 stars = 0.25 probability. 1 star = 0.0 probability. DFM simplifies and speeds up all the rating-scale computations to be those of the DM. In JMLE (as implemented in Winsteps and Facets), the

parameter estimation converges when “observed marginal score \approx expected marginal score” for all parameters. The expected marginal score is computed in the usual DM way. The observed marginal score is the sum of the prediction-probabilities (based on the Star ratings) for each parameter. The resulting DFM Pseudo-Rasch measures for movies and clients are effectively collinear with the RSM measures. The DFM model was implemented in special-purpose software. It achieved its objective of speeding up estimation without noticeably degrading prediction accuracy, relative to RSM.

5. Correction for extreme observations.

Since the Netflix criterion for accuracy of prediction was the RMSR, incorrectly predicting that an observation would be 1 or 5 Stars was considerably worse, on average, than incorrectly predicting that an observation would be in an intermediate category. The use of the extreme 1- and 5-Star categories by Netflix clients was somewhat idiosyncratic. An improvement to prediction resulted when the influence of the extreme categories was reduced. For the DFM model, experiments revealed that better inferences were obtained by substituting to 4.75 Stars (in place of 5 Stars) and 1.25 Stars (in place of 1 Star), and adjusting the probabilities accordingly. For the RSM model (as implemented in Winsteps), this is done by adjusting observed category frequencies. For 5 Stars, the observed rating-score was reduced from 5.0 to 4.75, and the corresponding observed category frequencies were changed from 1 observation of 5 into 0.75 observations of 5 and 0.25 observations of 4. Similarly for 1 Star.

6. Estimating RSM thresholds.

The estimation of RSM rating-scale thresholds has long been troublesome. The original JMLE technique, proposed in “Rating Scale Analysis” (Wright and Masters, 1982) estimated each threshold using Newton-Raphson iteration, as though it was an almost separate parameter. This technique proved too unstable when category frequencies were very uneven or there were pernicious patterns of missing-data. So Newton-Raphson iteration of the threshold estimates was replaced in Winsteps by “Iterative curve-fitting”, because the relevant functions are known to be smoothly monotonic logistic ogives.

For the Netflix data, a faster-converging estimation method for rating-scales was sought. An iterative approach based on solving simultaneous linear equations has proved effective. Suppose that P_k is the expected frequency of category k in the dataset according to RSM.

$$P_k = \sum_{n,i} P_{nik} = \sum_{n,i} \frac{e^{\sum_{j=0}^k K_{nij}}}{1 + \sum_{h=1}^m e^{\sum_{j=1}^h K_{nih}}}$$

where $K_{nij} = B_n - D_i - F_j$ except that $K_{ni0} = 0$. F_j is the Rasch-Andrich threshold at which categories $j-1$ and j are equally probable.

Suppose that a small change δF_j in F_j (and similarly for all the other thresholds) would produce the observed category frequency O_k :

$$O_k = \sum_{n,j} \frac{e^{\sum_{j=0}^k (K_{nij} - \delta F_j)}}{1 + \sum_{h=1}^m e^{\sum_{j=1}^h (K_{nij} - \delta F_j)}}$$

Then, since $e^{(x-\delta x)} \approx (1-\delta x)e$

$$O_k = \sum_{n,i} \frac{\left(1 - \sum_{j=0}^k \delta F_j\right) e^{\sum_{j=0}^k (K_{nij})}}{1 + \sum_{h=1}^m \left(1 - \sum_{j=1}^h \delta F_j\right) e^{\sum_{j=1}^h (K_{nij})}}$$

Then ignoring cross-products of the δ terms and since δF_0 does not exist:

$$O_k = \sum_{n,i} \frac{\left(1 - \sum_{j=1}^k \delta F_j\right) e^{\sum_{j=0}^k (K_{nij})} \left(1 + \sum_{h=1}^m \left(1 + \sum_{j=1}^h \delta F_j\right) e^{\sum_{j=1}^h (K_{nij})}\right)}{\left(1 + \sum_{h=1}^m e^{\sum_{j=1}^h (K_{nij})}\right)^2}$$

$$O_k = \sum_{n,i} \left(\left(1 - \sum_{j=1}^k \delta F_j\right) P_{nik} \right) \left(P_{ni0} + \sum_{h=1}^m \left(1 + \sum_{j=1}^h \delta F_j\right) P_{nih} \right)$$

$$O_k = P_k - P_k \sum_{j=1}^k \delta F_j + \sum_{n,i} \sum_{h=1}^m \left(\sum_{j=1}^h P_{nik} P_{nih} \delta F_j \right)$$

and similarly for the other categories, $k=1, m$.

At the end of each iteration, all the numerical values of the $\{O_k\}$, $\{P_k\}$ and $\{\sum P_{nik} P_{nih}\}$ terms are known. Consequently the $\{O_k\}$ equations become a set of simultaneous linear equations which can be solved for $\{\delta F_j\}$. Then $\{F_j + \delta F_j\}$ become the values of $\{F_j\}$ for the next iteration after standardization so that $\sum F_j = 0$. So far, this estimation technique has proved robust and fast.

7. Multidimensionality and Singular-Value Decomposition (SVD).

Multidimensionality is a serious threat to the validity of unidimensional Rasch measures. It also degrades the capability of the measures to predict observations. Single-parameter fit statistics (such as INFIT, OUTFIT and point-biserial correlations) are insensitive to pervasive multidimensionality. PCA of residuals is a useful tool for investigating multidimensionality, but it loses its power as the proportion of missing data increases, and the number of variables to be factored increases. With 99% missing data and 17,770 variables, PCA of residuals is almost ineffective. It does signal the existence of secondary dimensions, but not in enough detail to be useful for item selection or improved prediction.

SVD is mathematical technique that has been used for decomposing matrices into a bilinear form for over 130

years. It is robust against missing data and the size of the matrix to be decomposed, so it is ideal for this application. SVD was the first conspicuously successful multi-dimensional method used by Netflix competitors. Most of those applied it using raw-score models.

A first-level SVD model for the Netflix data, with SVD values $\{V_n\}$ for the clients and $\{U_i\}$ for the movies, is:

$$X_{ni} = E_{ni} + m(V_n * U_i) \pm \varepsilon_{ni}$$

where

$$E_{ni} = \left(\sum_{j=1}^m jP_{nij} \right)$$

for the Andrich Rating-Scale model.

Notice that Rasch residuals are explained, as far as possible, by two factors (U for movies and V for clients) which multiply together. The factor products center on zero, because the residuals sum to zero.

Maximum-Likelihood Estimation: the starting values of $\{U_i\}$ and $\{V_n\}$ are random uniform numbers [-1,1] and normalization after each iteration through the data is $Average(U_i^2) = 1$.

$$V'_n = V_n + \frac{\sum_{i=1}^L (X_{ni} - E_{ni} - m * V_n * U_i) m * U_i}{\sum_{i=1}^L k(mU_i)^2}$$

where k is chosen to prevent the iterative changes becoming too large.

There are now several options including:

A. The Rasch measures and the SVD values can be used to predict the Netflix Qualifying dataset. Further, a second-level SVD analysis can be performed on the residuals $\{\varepsilon_{ni}\}$ from the first SVD analysis, and then the Qualifying dataset again predicted this time from two levels of SVD. This process can be continued for more SVD factors. During the Netflix competition, better prediction was achieved down to about 40 SVD factors.

B. From a Rasch data-analysis perspective, the first-level SVD conveniently segments the data into quarters: positive and negative SVD values for the movies, positive and negative SVD values for the clients. There is also a central core in which the movie and client SVD values are too small to be meaningful. They are ratings to which the first-level SVD “dimensions” do not apply. The 5 data subsets identified by the first-level SVD can then be used in 5 more Rasch analyses, and Rasch measures generated for further prediction. This process can also be continued.

We can also perform this computation for the standardized residuals:

$$\frac{X_{ni}}{S_{ni}} = \frac{E_{ni}}{S_{ni}} + (V_n * U_i) \pm \varepsilon_{ni}$$

where

$$S_{ni} = \sqrt{\left[\left(\sum_{j=1}^m j^2 P_{nij} \right) - \left(\sum_{j=1}^m j P_{nij} \right)^2 \right]}$$

which is the model S.D. of the observation around its expectation for the Andrich Rating-Scale model.

$$V'_n = V_n + \frac{\sum_{i=1}^L \left(\frac{X_{ni} - E_{ni}}{S_{ni}} - V_n * U_i \right) U_i}{\sum_{i=1}^L k(U_i)^2}$$

8. The Perils of Overfit When the Data Used for Estimation are also Predicted.

RMSR computed from estimates based on Training (less Probe)				
Model	Training (less Probe)	Probe	Quiz	Test
Ratings	99,072,112	1,408,395	1,408,342	1,408,789
Andrich RSM	0.9139 Overfit	0.9858	0.9876	0.9867
Andrich RSM with 0.25 extreme correction	0.9167 Slightly less overfit	0.9853 Slightly better prediction	0.9871	0.9863
PCM	0.9147	0.9875	0.9897	0.9886

Netflix identified 1,408,395 ratings within the large Training dataset as the “Probe” dataset. They announced that the ratings in the Probe dataset had the same statistical characteristics as the ratings in the Quiz and Test datasets. Thus there were three similar datasets: the Probe dataset for which all the ratings were known, the Quiz dataset for which only the RMSRs for submissions were known, and the Test dataset about which nothing specific was known about the ratings. The Netflix Prize was awarded to the best prediction of the Test dataset.

The Probe dataset was only 1.4% of the Training dataset, but when it was included in the estimation process, the RMSR for the Probe data (predicted from the estimates) was noticeably lower than the RMSR reported for the Quiz data from the same set of measures. The inclusion of the data to be predicted within the estimation routine caused overfit to those data relative to the prediction of similar data not included in the estimation dataset.

Further, after each submission, an RMSR was obtained for the Quiz dataset. Over the months, this enabled a statistical picture of the Quiz dataset to be constructed which enabled some aspects of the prediction algorithms to be refined. The result was a slight overfit to the Quiz dataset. After the competition concluded, RMSRs for the

Test dataset were released. The winner's RMSR on the Test dataset was 0.8567, but slightly better, 0.8554, on the somewhat more public Quiz dataset. The minimal extra information about the Quiz dataset was enough to produce slight overfit to the Quiz dataset relative to the completely unknown Test dataset.

We can speculate about what the winning RMSR would have been if competitors had only been given the Training dataset and the list of ratings to be predicted. In this "real life" situation, competitors would receive no feedback about the success of their submissions or the statistical properties of the ratings to be predicted until the competition concluded. My guess is that the best RMSR would have been close to 0.90 instead of 0.86.

9. Amazingly Low Variance-Explained.

Analysts are sometimes perturbed to see the low percentage of the variance in the observations explained by the Rasch measures. According to RMT 20:1, 1045, www.rasch.org/rmt/rmt201a.htm - the variance-explained by the Rasch measures is often 50% or less. But can other models do any better? The S.D. of the Test dataset around its mean is 1.1291. The winning RMSR is 0.8567, so the variance explained by the winning submission after 34 months work is $(1.1291^2 - 0.8567^2) / 1.1291^2 = 42\%$, less than 50%, despite using the most elaborate statistical procedures available. In fact, one month after the competition started, on Nov. 2, 2006, the best RMSR was around 0.9042, this was $(1.1291^2 - 0.9042^2) / 1.1291^2 = 36\%$ variance-explained. In the other 33 months of the competition, only 6% more variance was explained despite Herculean efforts by an army of computer scientists equipped with a huge amount of computing power, all expended in an effort to win the Netflix Prize of \$1 million.

What does this mean in practice? The prize-winning RMSR was 0.8567. The mean-absolute-deviation is about $0.8 * \text{RMSR} = 0.8 * 0.8567 \approx 0.7$. So, if prediction of exact Star ratings is needed, even the best set of predictions can be expected to be errant by 1 or more Stars more often than they are dead on.

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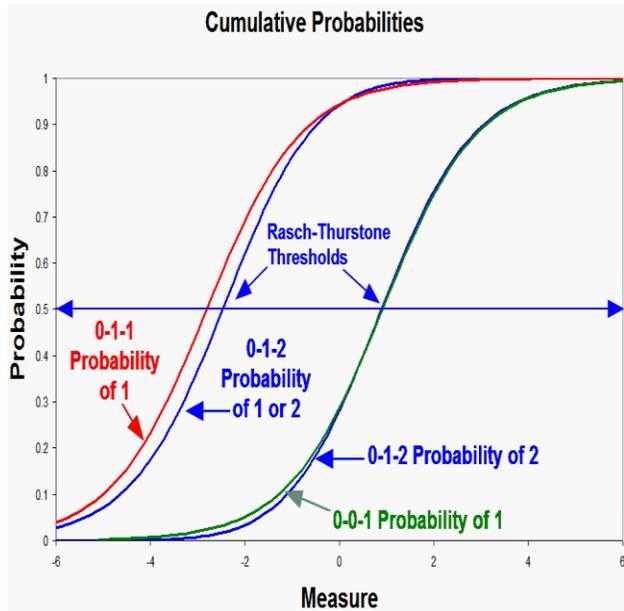
Rasch-related Coming Events

- Feb. 5, 2010, Fri. Proposal deadline: IOMW April 28-29 2010 International Objective Measurement Workshop, Boulder, CO, USA, www.iomw2010.net/
- Feb. 15, 2010, Mon. Deadline for Submission for June 7-9, 2010, Mon.-Wed. International Association for Computerized Adaptive Testing (IACAT) Conference, Arnhem, Netherlands, www.rcec.nl/iacat
- Feb. 22 - June 18, 2010, Mon.-Fri. Advanced course in Rasch Measurement online course (D. Andrich, I. Marais), www.education.uwa.edu.au/ppl/courses
- Feb. 23-25, 2010, Tues.-Thurs. Introducción al uso del Modelo de Rasch en Medición Educativa, Montevideo, Uruguay (C. Pardo, Spanish), email: Carlos Pardo, capardo -x- ucatolica.edu.co
- March 1-3, 2010, Mon. - Wed. Rubrics and Achievement levels in education and the use of Rasch Model (Winsteps), Santiago, Chile (C. Pardo, Spanish), Universidad Catolica Silva Henriquez, email: Susana Barrera, sbarrera -x- uchsc.cl
- March 5 - April 2, 2010, Fri.-Fri. Rasch - Further Topics (intermediate) online course (M. Linacre, Winsteps), www.winsteps.com/courses.htm
- March 10-12, 2010, Wed.-Fri. Introduction to Rasch (A. Tennant, RUMM), Leeds, UK, www.leeds.ac.uk/medicine/rehabmed/psychometric
- Apr. 28-29, 2010, Wed.-Thur. IOMW 2010 International Objective Measurement Workshop, Boulder, CO, USA, www.iomw2010.net
- Apr. 30 - May 4, 2010, Fri.-Tues. AERA Annual Meeting, Denver, CO, USA, <http://www.aera.net>
- Apr. 30 - May 28, 2010, Fri.-Fri. Rasch - Core Topics (introductory) online course (M. Linacre, Winsteps), www.winsteps.com/courses.htm
- May 12-14, 2010, Wed.-Fri. Introduction to Rasch (A. Tennant, RUMM), Leeds, UK, www.leeds.ac.uk/medicine/rehabmed/psychometric
- May 17-19, 2010, Mon.-Wed. Intermediate Rasch (A. Tennant, RUMM), Leeds, UK, www.leeds.ac.uk/medicine/rehabmed/psychometric
- June 7-9, 2010, Mon.-Wed. International Association for Computerized Adaptive Testing (IACAT) Conference, Arnhem, Netherlands, www.rcec.nl/iacat
- June 13-16, 2010, Sun.-Wed. International conference on probabilistic models for measurement in education, psychology, social science and health, Copenhagen, Denmark, conference.cbs.dk/index.php/rasch
- June 25 - July 23, 2010, Fri.-Fri. Many-Facet Rasch Measurement (intermediate) online course (M. Linacre, Facets), www.winsteps.com/courses.htm
- June 29 - July 1, 2010, Tue.-Thur. PROMS-KL 2010 Pacific Rim Objective Measurement Symposium Kuala Lumpur, Malaysia, www.iuu.edu.my/proms/2010

Dichotomizing Rating Scales

Findings based on rating scales can be difficult to explain to a non-technical audience. If the rating-scale categories can be bisected by a pass-fail cut-point, such as “agree or not”, “competent or not”, “impaired or not”, then it can simplify communication if the rating scale is dichotomized around the cut-point. Categories above the cut-point are scored “1”, and categories below the cut-point are scored “0”.

How do measures based on the dichotomized data relate to the measures based on the original ratings? The Figure illustrates the relationship.



The Figure shows a 3-category item, rated 0-1-2. The two cumulative probability ogives (based on the Rasch partial-credit model) for that rating scale are shown. The intersections of the ogives with 0.5 probability are the Rasch-Thurstone thresholds.

The rating scale can be dichotomized in two ways: 0-1-2 becomes 0-0-1 or 0-1-2 becomes 0-1-1. These two dichotomizations can be analyzed with the dichotomous Rasch model. To make comparison simpler, the person measures are anchored at their rating-scale estimates. The result of the dichotomous analysis is two dichotomous ogives, one for each of the two dichotomizations. The Figures indicates that the two dichotomous ogives approximate the cumulative probability ogives of the rating-scale analysis. Thus the difficulties of the dichotomized items approximate the Rasch-Thurstone cumulative-probability thresholds, not the Rasch-Andrich equal-adjacent-category-probability thresholds (which are generally more central).

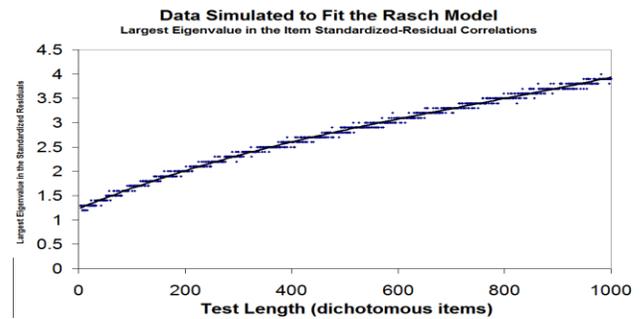
This result is reassuring because it indicates that inferences based on the simpler dichotomized data approximate inferences based on the more complex rating-scale data.

John Michael Linacre

More about Critical Eigenvalue Sizes in Standardized-Residual Principal Components Analysis (PCA)

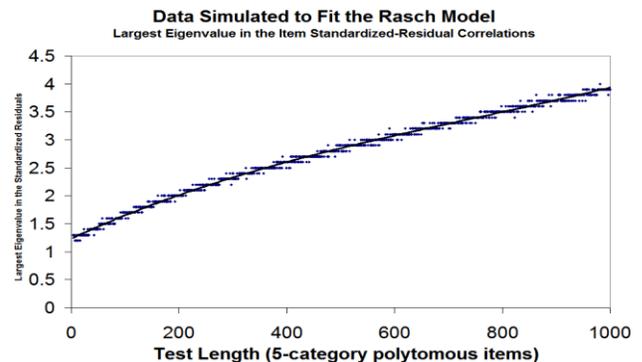
Gilles Raïche in [RMT 19:1 p. 102](#) reports eigenvalues in the range 1.4 to 2.1 for the first component in a PCA of inter-item correlations of standardized residuals of Rasch-fitting data. Test lengths were in the range 20 to 60 items.

Here those findings are extended to dichotomous and polytomous data with test lengths from 3 to 1000 items. The generating person sample of 1000 persons has a normal distribution with a mean of 0 logits and a standard deviation of 2 logits. The generating item distribution is uniform from -2 to +2 logits. For the 5-category polytomous data, the generating Rasch-Andrich thresholds are: -2.53, -0.35, 0.56, 2.32 logits. The Figures shows the eigenvalues sizes of the first components (contrasts) in a PCA of the standardized-residual item-correlation matrices.



For the dichotomous simulations, the eigenvalue increases from 1.3 for 3 items to 4.0 for 1000 items. For 5-category polytomous items, the eigenvalues have the same range.

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“Monte Carlo PCA for Parallel Analysis” is Marley Watkins’ free software at

www.public.asu.edu/~mwwatkin/Watkins3.html

for performing this type of investigation using simulated random-normal deviates, which standardized residuals approximate. For 200 items (variables) and 1000 persons (subjects), that software reports that the first PCA component in the random-normal deviates has an eigenvalue of 2.05 which accords with the findings above.

Alan Tennant